KEY ISSUES

What Do We Know About Lorentz Invariance?

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Abstract. The realization that Planck-scale physics can be tested with existing technology through the search for spacetime-symmetry violation brought about the development of a comprehensive framework, known as the gravitational Standard-Model Extension (SME), for studying deviations from exact Lorentz and CPT symmetry in nature. The development of this framework and its motivation led to an explosion of new tests of Lorentz symmetry over the past decade and to considerable theoretical interest in the subject. This work reviews the key concepts associated with Lorentz and CPT symmetry, the structure of the SME framework, and some recent experimental and theoretical results.

1. Introduction

Lorentz symmetry is a foundational assumption of both of our current best theories of physics: the Standard Model of particle physics and General Relativity. At the heart of Lorentz symmetry is the Principle of Relativity: the property of nature that experimental results do not seem to depend on the orientation of the laboratory (rotation invariance) or its velocity though space (boost invariance). The principle of relativity is a very old idea. In the context of mechanics, it can be traced back to at least the time of Galileo Galilei [1]. The Principle of Relativity requires that the laws of physics take the same form, independent of the velocity and orientation of the experiment.

During the years leading up to 1905, the Principle of Relativity received a great deal of attention. The mathematical transformation, now known as the Galilean transformation, which was thought to implement the change from one velocity to another did not appear to apply to the theory of electromagnetism as newly unified by Maxwell. A new transformation, the Lorentz transformation, found partly by Lorentz [2], was given its current interpretation by Einstein [3]. Einstein's interpretation forever changed our understanding of time, as well as other physical quantities such as energy, and in the process asserted that the Principle of Relativity applies to all phenomena. Lorentz symmetry is a global symmetry of the Standard Model in flat spacetime, and a local symmetry among freely-falling frames in General Relativity.

Though tests of Lorentz symmetry have been performed since the time of Einstein, the past several decades have seen considerable interest in the subject [4] and an explosion of new tests [5]. These tests have been performed across a wide range of fields of physics and have in some cases reached impressive sensitivity [5], though many ways in which Lorentz symmetry could be violated remain untested. The primary motivation for this resurgence of interest is the search for new physics at the Planck scale [6], though placing known physics on a sound experimental foundation also offers a motivation.

In the 1990s, a comprehensive effective field-theory based framework for studying Lorentz symmetry known as the gravitational Standard-Model Extension (SME) was developed [7]. The SME contains the Standard Model and General Relativity as well as all possible Lorentz-violating terms that can be constructed from the associated fields. Most of the recent high-sensitivity tests have been motivated and analyzed in this framework, and much theoretical work has also been done in this context. Hence the SME is the primary framework for discussion in this review. It should be noted that since the time of Einstein, a great deal of work has been done on the topic of Lorentz symmetry, far more than can be considered in this short review. The goal of this work is to review some key aspects and recent developments related to the effective field-theory approach to the search for Lorentz violation.

This paper is organized as follows. Section 2 presents some foundational material including a discussion of the role of symmetry in fundamental physics, comments on the relation of Lorentz symmetry to other symmetries, a simple example of rotation-invariance violation, and some additional discussion of the motivation for considering symmetry violation. The rational for studying the topic of Lorentz symmetry using the effective field-theory based framework provided by the SME is considered in section 3. Section 4 addresses the construction of the SME, presents some popular limiting forms, and reviews some theoretical studies of its structure. In section 5, a discussion of some key considerations associated with experimental and observational work on

Lorentz symmetry in the context of the SME is provided along with some discussion of the modern high-sensitivity tests that have been performed and the remaining space open to investigation. Motivated by current events and the possibility of explaining observed physics beyond the Standard Model, this section also provides some additional comments on neutrinos. Finally section 6 provides a summary and some conclusions regarding the current state of the field of Lorentz-symmetry study.

2. Basics

This section provides some basic information and simple examples useful in understanding the development to follow. We first consider symmetry in physics generically and the relation of Lorentz symmetry to other symmetries in section 2.1. We then consider rotation invariance as a simple example of both symmetry and of symmetry violation in section 2.2. Section 2.3 expands on the motivation for considering the violation of spacetime symmetries.

2.1. Symmetry

Symmetry in physics can be informally stated as a transformation on a system that leaves it unchanged. This approximately matches the everyday use of the word in which one might say that a sphere exhibits rotational symmetry since it looks the same when rotated. A physical symmetry then corresponds to a mathematical operation on the laws that describe the system such that there is no effect (up to coordinate choices) on the observable quantities associated with the system predicted by those laws. Symmetry has long been a guiding principle in physics, and in modern theoretical physics, it is given a lead role [8]. In proposing a theory, the desired symmetries are usually one of the foundational assumptions. A lagrangian is then written to contain all possible terms consistent with the symmetries and particles that have been assumed.

Lorentz invariance is an example of a symmetry in physics, which contains two subgroups: rotations and boosts. It is a spacetime symmetry since it is associated with transformations in the physical space. Other symmetries closely related to Lorentz symmetry include diffeomorphism invariance and the discrete symmetries, which include charge conjugation C, parity P, time reversal T, and combinations of these.

Of particular interest in the present context is the combination of all three discrete symmetries CPT due to its close connection with Lorentz symmetry as well as the fact that it is the only discrete symmetry or combination of discrete symmetries that is not violated in the Standard Model. There is a well-known result called the CPT theorem, which roughly states that local quantum field theories with Lorentz symmetry, including those used to describe known particle physics, also have CPT symmetry. For additional discussion of this result, see [9] and references therein. In 2002 a particularly strong form of the CPT theorem was proven by Greenberg [10]: "An interacting theory that violates CPT invariance necessarily violates Lorentz invariance." This implies that CPT violation consistent with the framework of conventional quantum field theory of known physics is described by the SME.

Note that throughout this work, references to Lorentz violation refer generically to the possibility of arbitrary violations of rotation or boost invariance. In particular, the existence of a preferred frame in which physics is isotropic is not assumed.

2.2. Rotation

Here we consider rotation symmetry, a part of Lorentz invariance, as a concrete example of symmetry. For simplicity, we consider the role of rotation symmetry in classical mechanics. We first consider a system with full rotation invariance. We then consider effective rotation-invariance violation as well as more fundamental rotation-invariance violation.

As an example of rotation invariance, consider a particle of mass m under the influence of a quadratic potential:

$$L = \tfrac{1}{2} m \dot{\boldsymbol{r}}^2 - \tfrac{1}{2} k \boldsymbol{r}^2, \tag{1}$$

where $\boldsymbol{r}$ is a position vector in 3 dimensions and k is a constant. Such a system could be constructed in a lab by attaching one end of a spring to a fixed point in a box (call it the origin) and attaching the other end to the particle. A rotation of this system can be carried out by instructing a worker in the lab to rotate the box. Mathematically this rotation can be carried out by applying an appropriate rotation matrix to the position vector, $r_{j'} = R_{j'k} r_k$. Explicit calculation will reveal that the lagrangian is unchanged and hence scientists in the lab will find identical observables associated with the rotated system for a given set of initial conditions.

In general, there are two types of Lorentz transformations that can by applied to a system: *Observer Transformations* and *Particle Transformations*. In the above example, we have performed a particle transformation, that is, we transformed, rotated in this case, the particles and fields involved in the experiment by instructing workers in the lab to physically rotate the experiment. An observer transformation in this case would involve choosing new coordinates. In the above example, which has rotation invariance, these two transformations can be chosen to have the same effect. This fact is a direct consequence of rotation invariance.

Next consider an example that has effective rotation-invariance violation. Consider a particle in a linear potential, an approximate model for a particle in an Earth-based laboratory, for example. Such a system may be mathematically described via the lagrangian,

$$L = \tfrac{1}{2} m \dot{\boldsymbol{r}}^2 - m \boldsymbol{g} \cdot \boldsymbol{r}, \tag{2}$$

where $\boldsymbol{g}$ is a constant background gravitational field and $\boldsymbol{r}$ is a position vector measured from some origin within some box in the lab. A rotation of the experiment in the lab, a particle rotation, could again be carried out by having a worker rotate the box, and such a transformation would be carried out mathematically by applying the rotation matrix to the position vector for the particle as before $r_{j'} = R_{j'k} r_k$. Since $\boldsymbol{g}$ is unaffected by this rotation of the box in the lab, the rotation matrix should not be applied to this vector. The lagrangian will not be invariant under such a transformation. The transformed lagrangian will take the form

$$L = \tfrac{1}{2} m \dot{\boldsymbol{r}'}^2 - g_k R_{kj'} r_{j'}. \tag{3}$$

This will have observable consequences. For example, in the unrotated system, the particle may fall toward a face of the box, where as in the rotated system it may fall toward a corner or a different face of the box.

I refer to the type of symmetry violation in the above example as "effective" because it is not a consequence of rotation-invariance violation in the fundamental laws of physics. It is instead a consequence of the fact that a true "rotation of the experiment" requires turning the Earth as well, since it is a part of this experiment. If

we did so, the rotation transformation would be applied to $\boldsymbol{g}$ as well, and we would see that the system is particle-rotation invariant. This idea of effective Lorentz violation has been applied to investigations of gravitomagnetism [11] and has been used to obtain some of the best constraints on spacetime torsion [12].

Note that an observer transformation could also be applied in the above example. This would involve rotating our coordinates in the lab. Under such a transformation, the rotation matrix would be applied to all vectors. The workers would make no adjustments to the physical system, and the particle would still fall toward the same face of the box. The only change would be in the name given to that direction. If the workers had originally called it the $\hat{x}$ direction, they might now call it the $\hat{y}$ direction.

As a final example, let us suppose that there is a hypothetical particle in our universe that experiences an acceleration of magnitude $\mathrm{a} = F/m$, when a force of magnitude F is applied in a given direction. Suppose further that the same particle experiences an acceleration of magnitude $\mathrm{a}' = F/m'$ when the force of magnitude F is applied in an orthogonal direction [13]. Assume that such an effect is found to be equally valid independent of the type of force, location in the universe, or other such factors. Such an effect would violate rotation invariance, but not due to the fact that some aspect of the experiment has been neglected as in the example of the gravitational field. It is an effect that could be attributed to the spacetime vacuum itself. A complete nonrelativistic theory having this feature can be written,

$$L = \tfrac{1}{2} m_{jk} \dot{r}_j \dot{r}_k - U(\boldsymbol{r}). \tag{4}$$

Here m_{jk} does not transform under particle transformations, but is a 2-tensor under observer rotations. This model can arise as a limit of the SME [13]. Here we have illustrated the basic idea of spacetime-symmetry violation using rotations due to the simple visual nature of this limit, however the same ideas apply to boost invariance and other spacetime symmetries.

2.3. Motivation

There are two common motivations for searching for Lorentz violation. Perhaps the most exciting is the possibility of detecting new physics at Planck scale with existing technology. Another motivation is provided simply by the desire to put a foundational principle of both of our current best theories, the Standard Model and General Relativity, on the strongest experimental grounds possible, since history demonstrates that deeply held principles are worth rigorous testing.

Though the Standard Model and General Relativity form an impressive description of physics at presently accessible energy scales, it is believed that they are merely the low-energy limit of a single quantum-constant theory at the Planck scale, 10^{19} GeV. Obtaining experimental information to guide the development of a theory of Planck-scale physics is notoriously difficult. For example, it is difficult to imagine probing this energy directly via particle-accelerator experiments as the Planck scale remains more than 15 orders of magnitude beyond the energy of the LHC.

An alternative approach is to search for Planck-suppressed deviations from known physics. The idea is to search for Planck-scale physics using Planck sensitivity rather than Planck energy. In fact, the realization that natural mechanisms for the generation of Lorentz violation exist in string theory [6, 14, 15], a leading candidate for the underlying theory, triggered much of the recent interest in Lorentz violation. Since that time, other scenarios for underlying theory compatible with Lorentz violation have

been found including ones based on noncommutative field theories [16, 17], spacetime-varying fields [18], quantum gravity [19, 20], random-dynamics models [21], multiverses [22], and brane-world scenarios [23].

Returning to the level of known physics, Lorentz symmetry is a foundational principle of both the Standard Model and General Relativity. This fact implies that the observation of Lorentz violation would be a strong indicator of new physics. The foundational nature of the idea also suggests that it should be placed on a strong experimental foundation. The history of physics suggests that broad testing of foundational assumptions, particularly those based on the beauty of symmetry, is well worthwhile.

The beauty of perfect symmetry is perhaps among the oldest ideas in physics and has perhaps even been given additional importance over time. Such ideas predate even modern science itself in Aristotle's assertion of perfectly-circular planetary orbits. As already noted, the Principle of Relativity in mechanics, as articulated by Galileo, was based on rotation invariance and Galilean-boost invariance. In modern theoretical physics, symmetries including Lorentz symmetry, the discrete symmetries, and gauge invariance are central. Although the beauty of perfect symmetry is appealing, historical examples of symmetry breaking are also prevalent. We are well aware that the planetary orbits are not perfect circles, every combination of the discrete symmetries is known to be broken except CPT, and electroweak-symmetry breaking is a key feature of the Standard Model.

A broad search appears to be a reasonable approach in searching for symmetry violation. For example, no test of parity symmetry with electromagnetic experiments, no-matter how sensitive, can exclude the possibility of parity violation in nature as parity violation is observed in weak-interaction physics but not electromagnetic physics. Such examples serve as motivation for a broad search for Lorentz violation.

3. A comprehensive theory: the SME

As noted in the examples in section 2.1, the classic approach to testing Lorentz symmetry is to do an experiment having a given orientation and boost, then repeat the experiment with a different orientation or boost and attempt to detect a difference. In principle this can be done randomly with any experiment. However proceeding in a random way has key disadvantages. First, there is no way of making quantitative comparisons between very different types of experiments. This makes it hard to answer questions about whether or not it is worth devoting resources to improve atomic-clock tests or whether one should do accelerator experiments instead. Second, it is often unclear which aspects of the experiment are most relevant to likely theoretical possibilities. For example, early CPT tests often averaged data taken over a period of time; however, since CPT violation comes with Lorentz violation, this averaging typically hides periodicity due to changes in orientation of the experiment. Hence proceeding without a comprehensive theory can lead to choices that mask a key signal. Finally, each theoretical model must confront experiment independently, with no easy way of cataloguing known constraints.

The above issues are addressed by the SME, which is a comprehensive field-theory based test framework in which to explore Lorentz violation. The framework consists of known physics in the form of the Standard Model plus General Relativity action along with all possible Lorentz-violating terms that can be constructed from the associated fields. This has been done explicitly for the case of power-counting renormalizable

mass dimension 3 and 4 operators in each sector of the Standard Model [7] as well as for the lowest-order terms in the gravity sector under the assumption of Riemann-Cartan geometry [24]. Higher mass-dimension operators have also been classified in several sectors [25, 26, 27]. Standard lore holds that the Standard Model and General Relativity are the low-energy limit of a unified theory at the Planck scale. Assuming a smooth match between physics at our current energies and Planck-scale physics, the Standard Model and General Relativity could be regarded as leading terms in a series approximation of the unified theory, with terms involving operators of higher mass dimension viewed as high-order corrections. The SME action provides the complete series for Lorentz-violating physics.

The key advantage of the SME is that it offers a complete field theory incorporating all possible forms of Lorentz and CPT violation that can be constructed from the associated fields. As a complete effective field theory, the SME provides complete predictive power. The outcome of any experiment can in principle be calculated. This allows one to predict *a priori* which experiments will attain the best sensitivity. It is possible to know which types of experiments measure the same thing and which measure different things, and such conclusions will be true in any consistent field-theoretic model. Thus it provides a framework for cataloguing results, forming a very definite list of constraints against which individual models can be compared. As a complete theory incorporating Lorentz violation, the SME is also very useful in considering theoretical aspects of Lorentz violation.

It is important to emphasize that the SME is not a model, rather it is a test framework designed for a broad search. Since the Standard Model and General Relativity have passed all experimental challenges thus far, the plan is to do a broad and comprehensive search for new physics rather than build specific models. The idea is that a more effective time for model building would be after a result is observed that is not consistent with the Standard Model and General Relativity, thus providing likely new physics to be modeled.

It should also be noted that a variety of other specialized formalisms exist that consider Lorentz violation. For a review of various approaches, see [28]. Examples that are consistent with effective field theory are contained in the SME framework. Section IV F of [25] and Section VI B of [27] demonstrate how several examples can be recast in the language of the SME. A common choice in specialized formalisms, including some constructed as special limits of the SME, is to consider models in which there exists an observer frame where rotation invariance is preserved, and Lorentz violation is associated purely with boost violation. This frame is often identified with the rest frame of the cosmic microwave background radiation, though other choices have also been considered. Such models are sometimes referred to as isotropic models, or with tongue-in-cheek as 'fried-chicken models' [29] in the literature. The latter name was coined by analogy with a popular food in the United States due to the popularity and simplicity of the models. The idea being that fried chicken is good, and everyone likes it, but if that is all one eats, one misses a lot. The frame in which physics is isotropic in such models is usually referred to as a preferred frame and associated effects are often referred to as preferred-frame effects. The term preferred-frame effects is sometimes used synonymously with Lorentz violation; however, this is not correct. In the SME the existence of a preferred frame is not assumed and generically the existence of Lorentz violation may not generate such a preferred frame. It should also be emphasized that the term isotropic is misleading, since even in isotropic models, physics is isotropic in only one frame. In all other frames, rotation invariance is

violated. Moreover, the isotropic frame cannot be chosen as the frame of an Earth-based laboratory, and rotation-invariance violation will be present in Earth-based experiments even in isotropic models.

4. Structure of the SME

The Lorentz-violating terms of the SME are constructed by coupling observer vector or tensor coefficients for Lorentz violation to Standard-Model operators. As an example, consider the following Lorentz-violating terms occurring in the quark sector:

$$\begin{aligned}\mathcal{L}_{\text{quark}}^{\text{CPT-even}} &= \tfrac{1}{2} i (c_Q)_{\mu\nu AB} \overline{Q}_A \gamma^\mu \overset{\leftrightarrow}{D^\nu} Q_B \\ &+ \tfrac{1}{2} i (c_U)_{\mu\nu AB} \overline{U}_A \gamma^\mu \overset{\leftrightarrow}{D^\nu} U_B \\ &+ \tfrac{1}{2} i (c_D)_{\mu\nu AB} \overline{D}_A \gamma^\mu \overset{\leftrightarrow}{D^\nu} D_B . \end{aligned} \tag{5}$$

Here capital Latin indicies are flavor indicies, while Greek indicies are spacetime indicies. The Standard Model covariant derivative is denoted D_μ, and the operation of the derivative on arbitrary objects A, B is defined as $A \overset{\leftrightarrow}{\partial_\mu} B \equiv A\partial_\mu B - (\partial_\mu A)B$. The notation

$$Q_A = \begin{pmatrix} u_A \\ d_A \end{pmatrix}_L \quad , \quad U_A = (u_A)_R \quad , \quad D_A = (d_A)_R \quad , \tag{6}$$

is used for the left- and right-handed quark multiplets, where $A = 1, 2, 3$ labels the flavor. The objects $(c_Q)_{\mu\nu AB}$, $(c_U)_{\mu\nu AB}$, and $(c_D)_{\mu\nu AB}$ are examples of coefficients for Lorentz violation. The size of these coefficients controls the amount of Lorentz violation of the given type in the theory. Note that each type of particle has its own coefficients for Lorentz violation, reflected here by the existence of the 3 coefficients shown explicitly each having flavor indices. Thus the generality of the SME allows for the possibility that Lorentz violation could exist in nature associated with one particle but not another. Such generality is important since history indicates that finding new physics requires looking in the right place as noted in section 2.3. The notation $\mathcal{L}_{\text{quark}}^{\text{CPT-even}}$ for the partial Lagrange density shown here indicates that it is the part of the lagrangian associated with the quarks containing coefficients for Lorentz violation that do not violate CPT symmetry. Other partial Lagrange densities exist in the theory that contain CPT-violating coefficients for Lorentz violation. The CPT even terms are easily recognized as they involve coefficients with an even number of spacetime indices, while CPT odd terms involve an odd number of spacetime indices.

The basic framework for including gravity in the general action-based construction of the Lorentz-violating SME was developed in 2004 [24]. Riemann-Cartan spacetimes, which allow for nonzero torsion in addition to curvature, were considered. Such spacetimes can be reduced to the Riemann spacetime of General Relativity in the appropriate limit. The systematic incorporation of Lorentz-violating operators in the gravitational-sector action was developed, and gravitational couplings were introduced in the other sectors of the theory. The consideration of Lorentz violation in nontrivial geometries led to startling theoretical conclusions regarding the compatibility of Riemann geometry with Lorentz violation as well as proposals for new types of experiments.

The remainder of this section addresses a number of key issues associated with the structure of the SME. A natural question at this stage is to ask where the coefficients

for Lorentz violation could come from and what other assumptions one might make about, for example, their spacetime dependence. Section 4.1 addresses this issue. Often a part of the full SME that is most relevant to a class of systems becomes the focus of a given study. A number of such limits of the SME are commonly used and these limits are reviewed in section 4.2. Many other theoretical issues associated with the structure of the SME have also been considered. Section 4.3 highlights the scope of the questions that have been addressed. A slightly older and somewhat more technical review of SME work is also provided by [30].

4.1. Symmetry breaking

The mechanisms by which coefficients for Lorentz violation could arise in the SME can be divided into two classes: explicit Lorentz-symmetry breaking and spontaneous Lorentz-symmetry breaking. Explicit Lorentz violation is characterized by directly assuming nonzero coefficients for Lorentz violation in the background, while spontaneous symmetry breaking assumes that the Lorentz violation arises dynamically as a Lorentz-violating solution associated with a Lorentz-invariant action. Here we provide some discussion of spontaneous breaking and some comparisons with explicit breaking.

Spontaneous symmetry breaking typically occurs when the low-energy solutions of a system do not respect a symmetry that exists in the theory. A classic example is provided by the case of a marble initially placed on the unstable equilibrium at the central high point of a Mexican hat. Any perturbation will cause the marble to drop to a lower energy configuration in the brim of the hat breaking the initial axial symmetry of the situation.

In the Standard Model, a potential analogous to the Mexican hat is used to give a vacuum expectation value to the Higgs field, spontaneously breaking $SU(2) \times U(1)$ gauge symmetry. The process fills the spacetime vacuum with a scalar condensate that affects fields coupling to the Higgs. The spontaneous breaking of a global symmetry results in massless Nambu-Goldstone modes, while the breaking of a local gauge symmetry results in massive gauge bosons, the W and Z bosons in the case of the Standard Model. Other particles coupling to the Higgs field also receive a mass related to the vacuum expectation value.

If the field that acquires a vacuum value is a vector or tensor object, then the spacetime becomes filled with a vector or tensor condensate, rather than a scalar as in the case of the Standard-Model Higgs. Here the original action has Lorentz symmetry, but the nature of the vacuum hides this symmetry at low energy. The couplings of such a field to Standard-Model fields generates the terms of the SME in a manner analogous to the development of mass for Standard-Model particles coupled to the Higgs field. Hence in flat spacetime, the implications of the couplings to the vacuum values associated with spontaneous Lorentz symmetry breaking are the same as when explicit breaking is assumed. However, additional phenomenology may arise due to both the Nambu-Goldstone [31] and massive modes [32] associated with Lorentz-symmetry breaking. For example, the long-range interaction of the associated Nambu-Goldstone mode can be identified with existing long-range interactions in nature, or must be interpreted as a presently undiscovered interaction. To date, models generating the photon in Einstein-Maxwell theory [31] and the graviton in General Relativity [33] as a consequence of Lorentz violation have been found. Other types of new interactions have also been considered [34, 35, 36].

A class of vector theories having spontaneous symmetry breaking known has bumblebee models were used to initiate the investigation of spontaneous Lorentz violation [37] and have recently been used to explore a variety of aspects of Lorentz-symmetry breaking [31, 32, 34, 38, 39]. Additionally, there is considerable early literature associated with couplings to vacuum-valued vector fields, references to which can be found in section III A of [32]. Similar models have also been used in a variety of contexts [40]. Beyond vector theories, spontaneous symmetry breaking in tensor theories has also been considered [33, 35], and some general results have been achieved [41].

While one is free to think of SME coefficients in flat spacetime as existing either by virtue of explicit or spontaneous Lorentz violation, incorporation of gravitation based on Riemann geometry requires one to consider spontaneous breaking [24]. The result, shown in the context of Riemann-Cartan geometry, forms a no-go theorem for explicit Lorentz breaking in gravity. The basic idea is that the structure of Riemann-Cartan geometry implies that the Bianchi identities of the curvature along with the field equations provide a constraint on the covariant conservation laws. This constraint is generally not satisfied for explicit breaking, signaling that such a theory is not self consistent. Conversely the constraint is automatically satisfied for spontaneous Lorentz violation due to the fact that the Lorentz-violating fields arise dynamically from within the theory. Reference [24] provides additional discussion and examples. The constraint that arises is developed explicitly in section 4.2.3 for the case of the leading-order Riemann limit of the pure gravity sector.

The no-go theorem for explicit Lorentz violation in gravity is a very significant result for several reasons. It provides one of the few opportunities to make a general statement about the nature and origin of possible Lorentz violation: if Lorentz-symmetry is violated, it must either occur spontaneously or a new geometrical framework for gravity must be found. As noted above, spontaneous breaking is distinguished from explicit breaking by the existence of modes related to fluctuations about the vacuum values in addition to the vacuum values themselves. The no-go theorem implies that consistent work with Lorentz violation in gravity requires consideration of the fluctuations in addition to the vacuum values associated with Lorentz breaking. This requirement makes gravitational phenomenology considerably more challenging. Model-independent methods of addressing the fluctuations have been achieved in phenomenological studies of the pure-gravity sector [38] as well as the gravitationally coupled matter sector [34]. The incompatibility of Riemann geometry and explicit Lorentz violation has also spurred consideration of Finsler geometry [42, 43]. Consideration of the geometric structure associated with SME coefficients has led to the discovery of new Finsler spaces known as SME spaces [42]. Other resolutions to the question of geometrical consistency may be possible if a suitable nongeometrical theory of gravity with Lorentz violation [44] were found. The no-go theorem also has implications for going beyond conventional theory [45].

4.2. Popular limits of the SME

The full SME includes an infinite number of operators of ever-increasing mass dimension. While this might seem daunting, several approaches have been used to keep the process tractable.

4.2.1. Minimal SME The most commonly used limit of the SME is the *minimal SME* [7], which has been studied extensively in the literature both theoretically and experimentally. This limit could be regarded as containing the leading Lorentz-violating terms of the series-approximation vision highlighted in section 3. In this limit, basically all of the usual properties of the Standard Model are maintained except particle-Lorentz symmetry and CPT symmetry. To be explicit, the following properties are maintained: the usual $SU(3) \times SU(2) \times U(1)$ gauge structure, power-counting renormalizability, energy and momentum conservation, $SU(2) \times U(1)$ symmetry breaking, quantization, microcausality, spin-statistics, and observer Lorentz covariance. Here maintaining energy and momentum conservation implies that attention is restricted to constant coefficients for Lorentz violation. That is,

$$\partial_\alpha t_{\mu\nu\lambda\ldots} = 0, \tag{7}$$

for any coefficient $t_{\mu\nu\lambda\ldots}$. This assumption is reasonable since it could be regarded as the leading term in a Taylor expansion of a coefficient with spacetime dependence. Additionally, many SME studies assume the coefficients for Lorentz violation are perturbatively small, based on the fact that Lorentz violation has not yet been seen in nature. Additional properties of specific coefficients are summarized in [5]. The assumptions of the minimal SME focus attention on one deviation from known physics, in this case Lorentz violation and its natural consequences. This reflects a principle frequently applied in SME studies, which is sometimes referred to as 'Kostelecký's Cutlass'. The principle states that no more than one deviation from known physics should be considered a time. The idea being that if ϵ is the probability that a given deviation from known physics provides a correct description of nature, then the probability of two such unobserved deviations being found together is order ϵ^2. Kostelecký's Cutlass can then be used to cut many such suggestions from consideration.

An associated limit is the gravitationally coupled minimal SME [24]. Here minimal couplings to gravity are considered in the same terms that appear in the flat-spacetime SME. In this context, asymptotically Minkowski spacetimes are often considered in which (7) is assumed to hold asymptotically, implying energy and momentum conservation asymptotically. However, geometrical consistency typically prevents the application of this condition beyond the asymptotic limit.

4.2.2. QED extension In much the way quantum electrodynamics (QED) can be extracted from the Standard Model, the QED extension can be extracted from the Standard-Model Extension [7]. The minimal QED extension is a popular limit studied extensively in the literature. The associated fermion lagrangian can be written

$$\mathcal{L}_\psi = \tfrac{1}{2} i \overline{\psi} \Gamma_\nu \overleftrightarrow{D^\nu} \psi - \overline{\psi} M \psi, \tag{8}$$

where

$$\Gamma_\nu \equiv \gamma_\nu + c_{\mu\nu}\gamma^\mu + d_{\mu\nu}\gamma_5\gamma^\mu + e_\nu + i f_\nu \gamma_5 + \tfrac{1}{2} g_{\lambda\mu\nu}\sigma^{\lambda\mu}, \tag{9}$$

and

$$M \equiv m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \tfrac{1}{2} H_{\mu\nu}\sigma^{\mu\nu}. \tag{10}$$

Here a_μ, ..., e_μ, f_μ, $g_{\lambda\mu\nu}$, and $H_{\mu\nu}$ are fermion-sector coefficients for Lorentz violation, ψ is the fermion field, and the γ^μ are the usual Dirac matrices. The covariant derivative in this context is now $D_\mu = \partial_\mu + iqA_\mu$, where A_μ is the photon field. Setting the

coefficients to zero results in the Lorentz-invariant limit reproducing the conventional Dirac lagrangian. It is common to treat mesons and baryons as having independent coefficients for Lorentz violation in this context. Though difficult in practice, these coefficients could be expressed in terms of the quark and gluon content of the particle.

The photon lagrangian takes the form

$$\begin{aligned} \mathcal{L}_A = & -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} \\ & + \tfrac{1}{2}(k_{AF})^{\kappa}\epsilon_{\kappa\lambda\mu\nu}A^{\lambda}F^{\mu\nu} - (k_A)_{\kappa}A^{\kappa}, \end{aligned} \tag{11}$$

where the coefficients for Lorentz violation here are $(k_F)_{\kappa\lambda\mu\nu}$, $(k_{AF})^{\kappa}$, and $(k_A)_{\kappa}$, and $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength. A helpful analogy to the effect of the coefficients for Lorentz violation is provided by electrodynamics in anisotropic and gyrotropic media [7]. Note also that the Chern Simons term [46], which continues to receive considerable attention [47] is contained here in the k_{AF} term.

As a means of providing a sense of the number of coefficients for Lorentz violation involved a typical SME analysis, we consider the number of independently observable coefficients associated with ordinary matter (protons, neutrons, and electrons), in the nonrelativistic flat-spacetime limit of the QED extension. Here there are 132 independently observable coefficients [5]. Based purely on a counting of coefficients, this limit is comparable in complexity to the number of free parameters in the minimal supersymmetric model, another popular deviation from known physics. Note however, that the spirit of the coefficients for Lorentz violation in the SME is to provide a broad catalogue of all possible deviations from Lorentz symmetry, which is rather different from the free parameters of a specific model such as supersymmetry.

To gain further intuition about the coefficients for Lorentz violation, consider two examples. First, in the newtonian limit, the ¸coefficient generates the direction-dependent effective inertial mass [13] considered in section 2.2 with

$$m_{jk} = m(\delta_{jk} + c_{jk} + c_{kj}). \tag{12}$$

The implications of ¸at any scale are similar, altering the relation between speed and energy as well as velocity and momentum, and doing so in a direction-dependent way.

As another simple example, consider the coefficient ¸ having an axial vector coupling to the fermion field above. The leading effect of this coefficient at the nonrelativistic level [48] is the contribution

$$H_{\text{Non Rel}} \supset \boldsymbol{b}\cdot\sigma, \tag{13}$$

where $\boldsymbol{b}$ is the spatial content of ¸ This term leads to a precession of spins [49], an effect that has been studied extensively, which reflects the violation of angular-momentum conservation that accompanies Lorentz violation.

The gravitationally coupled minimal QED extension has also been considered [24, 34]. Here minimal couplings to gravity are considered in the same terms appearing in Eqs. (8) and (11). The vierbein formalism is used for the incorporation of fermions and the derivatives become covariant derivatives for spacetime as well as $U(1)$. As an example, the term associated with the ¸coefficient takes the form

$$\mathcal{L}_\psi \supset \tfrac{1}{2}iee^{\mu}{}_{a}\overline{\psi}c_{\alpha\beta}e^{\beta a}e^{\alpha}{}_{b}\gamma^{b}\overleftrightarrow{D}_{\mu}\psi, \tag{14}$$

where $e_{\mu}{}^{a}$ is the vierbein and e is the vierbein determinant. As is standard in SME studies with gravity, Greek indices refer to general spacetime coordinates, while Latin indices refer to local Minkowski coordinates.

4.2.3. Leading-order Riemann limit A key condition of the minimal SME is power-counting renormalizability, a property already absent from conventional General Relativity. The analogous limit in the gravity sector is the restriction to operators of no higher mass dimension than those of the conventional Einstein-Hilbert action. Such terms are referred to as leading-order terms. A further restriction that is natural in many circumstances is the Riemann limit in which torsion vanishes. This limit is relevant, for example, in solar-system phenomenological studies where torsion effects are typically suppressed making coupling of coefficients for Lorentz violation to torsion of considerably lesser interest than couplings of Lorentz violation to curvature. It is also common to specialize further to the limit of linearized gravity [38, 50].

Considerable work on Lorentz violation in gravity has also been done in the context of the parametrized post-newtonian (PPN) formalism [51]. The philosophy of the PPN is somewhat analogous to the philosophy of the SME; however, their goals and methods are rather different. Reference [38] provides a detailed comparison between the PPN and the leading-order Riemann limit of the SME. Here we summarize several key points. As a comparison of motivation, that of the PPN is to provide a test framework parameterizing deviations from General Relativity, some of which are Lorentz violating, while the SME parametrizes deviations from exact Lorentz symmetry, some of which result in deviations from General Relativity. In terms of methods, the PPN provides an expansion about the General Relativity metric, while the SME provides an expansion about the action of General Relativity and the Standard Model. In terms of Lorentz violation, the PPN assumes that physics is isotropic in a particular frame, while the SME makes no such assumption. Perhaps most interestingly, reference [38] finds only a one degree of freedom overlap between the PPN and the leading-order Riemann limit of the SME, implying that the SME provides many new opportunities for existing gravitational experiments.

Since the pure-gravity sector is an area where the no-go theorem of section 4.1 plays an especially prominent role, we conclude this section by illustrating its implications in the leading-order Riemann limit of the SME. The pure-gravity action in this limit takes the form

$$S_{\mathrm{gravity}} = \frac{1}{2\kappa}\int d^4x[e(1-u)R - 2e\Lambda + es^{\mu\nu}R_{\mu\nu} + et^{\kappa\lambda\mu\nu}R_{\kappa\lambda\mu\nu}]. \tag{15}$$

Here $1/2\kappa \equiv 1/16\pi G_N$, where G_N is Newton's constant, R is the curvature scalar, $R_{\mu\nu}$ is the traceless Ricci tensor, $R_{\kappa\lambda\mu\nu}$ is the Weyl tensor, Λ is the cosmological constant, $s^{\mu\nu}$ and $t^{\kappa\lambda\mu\nu}$ are coefficient fields for Lorentz violation, and u is a coefficient field though not Lorentz violating. Variation of the action with respect to the metric $g_{\mu\nu}$ yields a modified Einstein equation of the form

$$G^{\mu\nu} - (T^{Rst})^{\mu\nu} = \kappa T_g^{\mu\nu}. \tag{16}$$

Here the material on the left comes from variation of the partial action S_{gravity} from equation (15), where $G^{\mu\nu}$ is the usual Einstein tensor and $(T^{Rst})^{\mu\nu}$ contains the additional material associated with the coefficient fields in equation (15). In a theory having spontaneous breaking, there will also be a partial action associated with the dynamics of the coefficient fields that may contribute energy momentum to the term on the right-hand side of (16). In the present context of pure gravity only, this will be the only contribution to the right-hand side. Acting on (16) with D_μ and using the trace Bianchi identity $D_\mu G^{\mu\nu} = 0$ yields

$$0 = D_\mu(\kappa T_g^{\mu\nu} + (T^{Rst})^{\mu\nu}), \tag{17}$$

imposing a constraint on the coefficient fields contained within $(T^{Rst})^{\mu\nu}$. The trace Bianchi identity stems purely from geometry, resulting in a geometric constraint on the energy momentum tensor that arises through variation with respect to $g_{\mu\nu}$. This constraint is not in general satisfied for explicit Lorentz-symmetry breaking, but is automatically satisfied for spontaneous-symmetry breaking. This can be verified explicitly in specific models. Additional discussion and examples appear in reference [24].

4.2.4. Nonrenormalizable operators A natural way to go beyond the minimal SME is to relax the condition of power-counting renormalizability, and hence consider Lorentz-violating operators of arbitrary mass dimension. This allows consideration of the full Lorentz-violating series approximation of the underlying theory. These higher-dimension operators might be expected to be particularly relevant in very high energy processes. Models containing Lorentz violation also exist in which higher-dimension operators provided the leading [16] or dominant Lorentz-violating effects [52, 53].

Nonrenormalizable operators of arbitrary mass dimension have been considered explicitly and comprehensively in the pure photon sector [25] and in the fermion sector [26, 27]. More specialized work on higher-dimension Lorentz violation has also been done [20, 54, 55]. In the same spirit as the minimal SME, references [25, 26, 27] consider nonrenormalizable Lorentz-violating terms that maintain most of the other usual properties of the Standard Model. To do so, they focus on operators that are quadratic in the conventional fields and that maintain conservation of energy, momentum, and electric charge. The subset of operators having these properties can typically be written in a form similar to terms in the minimal SME in which the objects appearing in a manner analogous to the minimal-SME terms become differential operators.

Here we consider the infinite set of CPT-odd coefficients of mass dimension d in the photon sector as an example of the above structure. The contribution of these terms to the lagrangian takes the form

$$\mathcal{L} \supset \tfrac{1}{2}\epsilon^{\kappa\lambda\mu\nu} A_\lambda (\hat{k}_{AF})_\kappa F_{\mu\nu}, \tag{18}$$

where

$$(\hat{k}_{AF})_\kappa = \sum_{d=\mathrm{odd}} (k_{AF}^{(d)})_\kappa{}^{\alpha_1 \ldots \alpha_{(d-3)}} \partial_{\alpha_1} \ldots \partial_{\alpha_{(d-3)}}. \tag{19}$$

The sum here ranges over values $d \geq 3$. Note that (18) differs from (11) only by the hat above the $(\hat{k}_{AF})_\kappa$ indicating that it is now a differential operator rather than a constant coefficient as in the minimal case. Note also that the minimal case is recovered in the limit $d = 3$.

The infinite number of coefficients studied in the nonrenormalizable limit are further tamed by a classification scheme based on the rotation properties of the coefficients. The method results in an expansion in terms of spherical coefficients for Lorentz violation and spin-weighted spherical harmonics. Such a decomposition is well suited to many experimental scenarios, which take advantage of the rotation properties of the coefficients.

4.3. Theoretical work

A large amount of work has been done on various theoretical properties of the SME. Though space prohibits a full consideration of many of these fascinating topics, this

section briefly highlights some of the areas that have been explored and provides references for the interested reader.

4.3.1. Observability of coefficients In a few cases, coefficients for Lorentz violation appearing in the general SME expansion are unobservable and can be removed from the theory by field or coordinate redefinitions. As one simple example of the required style of thinking, consider the a_μ term appearing in the fermion sector (8). The field redefinition, $\psi(x) = \exp[ia_\mu x^\mu]\chi(x)$, will remove this term from the theory in the single-fermion Minkowski-spacetime limit [7]. Note that this is a field redefinition and not a gauge transformation, as the field A_μ is uninvolved. If one instead chooses to work with the original field ψ, a_μ terms will remain in relevant calculations, but will not lead to observable effects in the single-fermion Minkowski-spacetime limit. The a_μ coefficient can be observed outside of this limit. References [7, 24, 25, 27, 34, 56, 57, 58] consider various other instances in which coefficients can be removed from the theory.

4.3.2. Stability and causality The questions of stability and causality in the SME were first considered in detail in Ref. [59]. This work considers the question in the context of the single-fermion limit of the free-matter sector of the SME. It is found that difficulties with stability or causality generally arise in theories having explicit Lorentz violation; however, if the coefficients are Planck suppressed, as expected in the typical motivation for the study of Lorentz violation, the difficulties occur at high energies or high boosts only. In this regime, the validity of low-energy effective field theory might be questioned anyway. This work also shows explicitly that spontaneous Lorentz violation in suitable scenarios can avoid these problems. In these scenarios, nonrenormalizable terms play a key role as energies approach the Planck scale, perhaps providing some bottom-up motivation for viewing the SME as a series approximation for Planck-scale physics, and providing support for the stability and causality of the SME as a theory emerging at low energies from spontaneous breaking in a realistic string theory.

Work on stability and causality has also been done in the context of other areas of Lorentz violating quantum field theory [60]. For example, a scalar quantum field theory with Lorentz violation has been shown to have reasonable stability and causality properties [61].

4.3.3. Renormalization While the minimal SME contains operators that are power-counting renormalizable, the details of renormalization have been the subject of considerable work. Here we highlight some key results and demonstrate the breath of work that has been done. Early work demonstrated the 1-loop renormalizability of the QED extension [62]. Renormalizability at 1 loop has since been shown for pure Yang-Mills theory with Lorentz violation [63], a result subsequently extended to include fermions, along with some additional generalization of results, to show the explicit 1-loop renormalizability of the gluon sector of QCD with Lorentz violation [64]. The 1-loop renormalizability of the electroweak sector of the SME has also been investigated [65]. Renormalization in Lorentz-violating QED beyond 1 loop [66], and in a curved background [67] have been considered, as have topics including scalar and Yukawa field theories [68] and nonpolynomial interactions [69].

In addition to establishing basic features of the field theory, the associated question of the running of the coefficients for Lorentz violation is relevant to

understanding the role of Lorentz violation across energy scales [62].

A variety of other quantum field-theoretic properties have also been explored in the context of Lorentz violation. For example, the following areas have been explored: radiatively induced Lorentz violation [70], Yang-Mills instantons [71], properties of the modified Dirac equation [72], the connection between noncommutative field theory and the SME [73], the Källén-Lehmann representation for Lorentz-violating field theory [74], Gupta-Bleuler photon quantization [75], massive photon theory [76], Yukawa-type quantum field theory with Lorentz violation in the bosonic sector [77], theories with nonpolynomial interactions and spontaneous Lorentz violation [78], and the implications of spontaneously breaking gauge symmetry with Lorentz violation [79].

4.3.4. Modified reactions Beyond technical questions associated with the SME as a field theory, considerable work has been done associated with calculating the nature of modifications to various high-energy processes. Here we list some examples of work in this area that the interested reader may wish to explore.

Early work on the SME established some general features of cross sections and decay rates [80]. In the usual analysis of some processes, Lorentz invariance is used in developing the necessary tools. Some of the alternative procedures needed were developed in this work. Electron-positron pair annihilation into two photons was considered as a specific example. Some other examples of modified processes considered in the literature include: Compton scattering [81], synchrotron radiation [82], Lorentz-violating effects on the thresholds of various processes [83], the possibility of photon splitting in Lorentz-violating field theory [84], modifications to α-decay associated with the composite coefficients for Lorentz violation of the α particle [85], and the effects of CPT violation on baryogenesis [86]. In some cases, processes forbidden in conventional physics become allowed. Vacuum Čerenkov is one example that has been considered fairly extensively in the literature [87]. The effect becomes allowed as charged particles may exceed the vacuum speed of light in the presence of some coefficients for Lorentz violation. There is a strong analogy here with conventional Čerenkov radiation in materials.

4.3.5. Supersymmetry Connections with other symmetries have been studied including various investigations of connections with supersymmetry. In reference [52], it was established that Lorentz-violating supersymmetric field theories exist. This was illustrate with simple examples related to the Wess-Zumino model. There is also a philosophical connection between supersymmetry and Lorentz violation highlighted in that work. Supersymmetry is a hypothesized spacetime symmetry. If it exists in nature, experiment suggests that it must be broken, and spontaneous breaking is an attractive mechanism. This situation seems somewhat in parallel with the present discussion of the possible breaking of the spacetime symmetries of Lorentz and CPT symmetry. The interested reader can find more information about supersymmetry and Lorentz violation in references [53, 88]. As noted in section 4.2.4, another interesting feature of some supersymmetric models is the generation higher mass-dimension Lorentz-violating terms as the dominant Lorentz-violating contribution [53].

4.3.6. Classical and nonrelativistic limits While the SME is a relativistic quantum field theory, various limits such as the classical or nonrelativistic limits yield additional theoretical understanding and are useful for many phenomenological investigations.

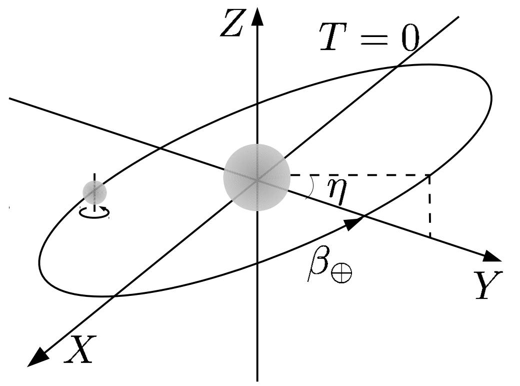

Figure 1. Sun-centered frame [5].

Various aspects of these limits have also received considerable attention [13, 48, 89]. Examples of these uses have been considered above in Eqs. (12) and (13).

5. Experiments and observations

This section addresses basic aspects of searches for Lorentz violation in the SME, summarizes the types of experiments that have been done, and offers some comments on current constraints.

5.1. Standard frame

Under an observer Lorentz transformation, the form of the coefficients for Lorentz violation changes. For example, an observer boost by β in the x direction of some observer frame results in

$$\begin{aligned} b_{0'} &= \gamma(b_0 - \beta b_x) \\ b_{x'} &= \gamma(b_x - \beta b_0). \end{aligned} \tag{20}$$

Hence constraints on b_0 and b_x obtained in the original observer frame will appear as mixed and scaled constraints on $b_{0'}$ and $b_{x'}$ when reported in the new frame. Additionally, the coefficients for Lorentz violation will become time dependent in a frame that is constantly rotating or changing its boost.

To address these issues in a manner most relevant for experiments on or near Earth, the Sun-centered celestial equatorial frame has been chosen as the standard frame for reporting measurements of coefficients for Lorentz violation [90]. Figure 1 illustrates this frame. The vernal equinox in the year 2000 is chosen as the origin of the time coordinate, T. The Z axis is parallel to the rotation axis of the Earth at $T = 0$, the X axis points from the Sun toward the vernal equinox, and the Y axis completes the right-handed coordinate system. Note that the choice of capital letters for the coordinates in this system is standard in the literature. Here $\beta_\oplus$ is the boost velocity of the Earth in the Sun-centered frame having a value of approximately 10^{-4} and $\eta \approx 23.5^o$. Additional information about this frame including transformations to other standard frames can be found in Section III A and Appendix C of [56], while implications of the frame choice in a gravitational context are considered in [34, 38].

This choice of standard frame is useful since it is approximately inertial over the time-scales of most relevant experiments, and it is convenient for the majority of tests performed on Earth and in the solar system. It is also worth noting that this frame is chosen simply to provide a standardized choice for reporting constraints. This frame is not a preferred frame as the term is often used in the literature, nor are any other special assumptions made about it.

5.2. Experimental and observational signatures

In searching for, or attempting to constrain Lorentz violation, experiments and observations typically take advantage of one or more of the following basic ideas: classic boost or orientation dependence, particle species dependence, CPT testing, or modified processes.

The experimental or observational hallmark of Lorentz violation is a result that changes when the experiment is repeated in a new inertial frame having a new orientation or new velocity. In the original Michelson-Morley experiment, as well as modern-day improvements, this would correspond to a fringe shift as the interferometer is rotated in the lab. Another modern test of Lorentz symmetry that provides a popular example is to search for a variation in the tick rate of a clock as the system is rotated or boosted [91]. Many modern experiments use rotation in the lab, but many more take advantage of the rotation of the experiment provided by the Earth over a sidereal day. Boost invariance can also be tested as experiments in the lab change velocity do the Earth's sidereal rotation and annual revolution. Rotating satellites and those with higher boost factors can provide even more advantages [34, 90].

Other types of signatures take advantage of the possibility of particle-species dependent coefficients for Lorentz violation. As one example, effective Weak Equivalence Principle violation results when terms involving particle-species dependent fermion-sector coefficients are coupled to gravity. This results in novel signals in Weak Equivalence Principle tests that come with the characteristic boost and orientation dependence of Lorentz violation [34].

Another key signal arises in comparing particles and antiparticles. As discussed above, coefficients with odd numbers of indices are CPT odd while those with even numbers of indices are CPT even. This implies different predictions for antiparticles over particles as CPT odd terms change sign. Under special circumstances tests with antimatter can yield novel sensitivities to coefficients for Lorentz violation [34]. As an example, differences in the spectrum of hydrogen and antihydrogen would be a signal of CPT violation, which would also come with the characteristic boost and orientation dependence of Lorentz violation [92].

Finally, the additional terms included in the SME can lead to modifications to various physical processes, which would also come with boost and orientation dependence. A simple example is provided by the rotation-invariance violation that enters Newton's second law from SME coefficient ¸discussed in section 2.2. The orientation dependent effective mass here leads to accelerations in directions in which there is no force. As a more exotic example, Lorentz violation in the photon sector can lead to vacuum birefringence. Searching for such possibilities often offers the possibility of impressive observational sensitivities.

5.3. General considerations

Before discussing sensitivities achieved via the above methods, it is worth considering a few general aspects of the search for Lorentz violation. First, it should be noted that there is really no such thing as a single "best test of Lorentz symmetry" or "best test of CPT symmetry". Imagine the analogous statement about parity symmetry prior to the discovery of parity violation. Claiming the best test of parity in electromagnetic interactions is irrelevant in claiming that other interactions are parity invariant and such a test does nothing to help detect the parity violation that does exist in nature. Similarly, it is not really possible to make particularly compelling statements about which coefficients for Lorentz violation are most likely to be nonzero, or which experiments have the best probability of finding Lorentz violation. A constraint of 10^{-40} with photons [5] does not exclude a signal at 10^{-10} in gravity.

It is also tempting ask at what level one might expect to find Lorentz violation originating from the Planck scale. Any attempt at a definitive answer to this question would likely be highly model dependent and at odds with the philosophy of a broad search. However, dimensional arguments provide some sense of scale. For example, in the case of a dimensionless coefficient, one might imagine Planck-suppressed effects generating nonzero values on the order of the ratio of an energy scale associated with the relevant physics to the Planck mass. Hence one could consider, for example, the mass of the electron over the Planck mass and arrive at 10^{-23} or perhaps at the larger extreme, the electroweak scale over the Planck scale and arrive at 10^{-17}.

5.4. Current limits

To date, no compelling evidence for Lorentz violation has been found. Thus this section focuses on current limits and large areas of open space for exploration. Current limits on coefficients for Lorentz violation are tabulated and updated annually in the publication *Data Tables for CPT and Lorentz Violation* [5]. At the time of writing this review, there are over 1000 published limits tabulated in that work (including multiple improvements in sensitivity to the same coefficient). Though it is not appropriate, nor is there space, to repeat all of that material here, this subsection will summarize the breath of measurements that have been made, the impressive sensitivity has has been reached in some cases, and the large unconstrained regions of coefficient space.

Sensitivities have been achieved with a large number of particles using a wide variety of physical systems. These include sensitivity to coefficients in the minimal SME associated with electrons [46, 91, 93, 94, 95, 96, 97, 98, 99], protons [91, 93, 94, 100, 101], neutrons [91, 93, 95, 100, 102, 103], photons [25, 56, 57, 96, 104, 105, 106, 107], muons [97, 108], taus [97], neutrinos [26, 109, 110, 111], quarks [97, 112], the Higgs [113], the W-boson [97], and the gluon [104]. Coefficients associated with protons, neutrons, and electrons have also been sought using gravitational couplings [11, 34, 98, 114] and tests have been used to place constraints on coefficients associated with the gravitational field in the leading-order Riemann limit [38, 115]. Nonrenormalizable coefficients associated with photons [25, 105, 106, 116], neutrinos [26], and other fermions [27] have also been investigated experimentally and observationally. Fermion results based on a number of works [117] are tabulated in [27]. In addition to the sensitivities noted above, considerable phenomenological work suggesting tests has also been done. Examples of these proposals are contained within many of the references noted above that contain some constraints as well

as in a number of theoretical references. Here we point the reader to several key phenomenological works in various areas as well as to some phenomenological analysis not cited elsewhere in this work: photons [25, 54, 56, 105, 118]; space-based tests [90, 119]; neutrinos [26, 29, 120, 121, 122]; ordinary-matter tests such as clocks, particle motion, etc. [27, 34, 91, 123]; antimatter [34, 92, 124]; mesons [125]; and gravity [34, 38, 126].

If the arguments of section 5.3 provide a reasonable guide, experiments are in some cases approaching the astounding sensitivity required to probe effects at the level of 2 Planck suppressions and some observational sensitivities have exceeded this level. For example, sensitivity to the _coefficient for the neutron has reached the level of 10^{-32} GeV in comagnetometer experiments [100, 102] and sensitivities to $\boldsymbol{k_{AF}}$ in the photon sector have reached the level of 10^{-43} GeV via CMB polarization analysis [25, 57, 105]. At the other end of the spectrum, large regions of coefficient space remain open, with no explicit observational or experimental constraints. For example, only one constraint exists associated with the tau lepton, and even in the context of minimal coefficients associated with ordinary matter, one of the most accessible places for experiments, dozens of coefficients remain unconstrained. Thus a large number of ways Lorentz symmetry could be violated remain unexplored. In some cases Lorentz violation could still be comparatively large and yet have evaded detection to date, a possibility known as countershaded Lorentz violation [34]. Matter-sector coefficients that are observable only when coupled to the gravitational field [34] and neutrino-sector coefficients observable only in processes involving neutrino phase-space properties [121] are explicit examples. In such cases, Lorentz violation can be large enough that the Lorentz hierarchy problem [15] can be obviated.

As a final note, although no compelling evidence for Lorentz violation has been found, it is worth noting that one cannot quite call all of the experimental and observational work that has been done "limits" since a few tests find nonzero values at the level of a few sigma.

5.5. Neutrinos

Neutrinos deserve special comment in the context of Lorentz violation for several reasons. First, neutrinos are perhaps the only area in which physics beyond the Standard Model is seen quite definitively in the form of neutrino oscillations. While massive neutrinos are a natural way to explain the observed oscillations, the frequently-heard statement that oscillations imply massive neutrinos is not correct. In fact it is possible to generate oscillations in SME-based models with Lorentz violation and massless neutrinos [29, 109, 127]. Second, several pieces of experimental evidence do not fit within the 3 mass model such as the MiniBooNE anomaly [128], the LSND anomaly [129], and CPT asymmetries of the MINOS type [130]. These effects can be accommodated in some SME-based models [26]. Other SME-based models for neutrinos have also been developed [120, 131] having a variety of features. For a general discussion of classes of models, see Section V of [26]. In addition to efforts to build models with Lorentz violation as alternatives to the standard 3 mass model, one can also consider Lorentz violation as a perturbing effect on the standard 3 mass model.

Finally, the possibility of neutrinos as faster-than-light particles has been around for some time [132] and is an effect that can arise due to nonzero SME coefficients for Lorentz violation. The idea received considerable attention due to the recent

OPERA result indicating such an observation [133], now believed to be a systematic [134], of neutrinos traveling faster than the speed of light. At first glance, this was perhaps the most exciting piece of experimental information to arise in the field of Lorentz violation. Such a result would have provided clear evidence of Lorentz violation interpretable as nonzero SME coefficients for Lorentz violation. On closer inspection, the result had several surprising features that perhaps further increased the degree of skepticism that such a paradigm-changing result naturally would receive. First, the effect was quite large, with neutrinos appearing to exceed the speed of light by 10^{-5} [133], an effect that could occur with, for example, dimensionless SME coefficients of the same order [26]. While it has been shown that Lorentz violation of this size could certainly evade detection in special circumstances [34], the effect is large compared with Planck suppression as well as many existing limits. The size of the signal also made it more challenging to incorporate the result alongside other time of flight measurements and other data on Lorentz violation in neutrinos [5]. Perhaps more interestingly, one would typically expect vacuum Čerenkov radiation to occur for faster than light particles [87] that would have an effect on the energy spectrum of the neutrinos; however, such an effect was not seen [110]. If such a signal were observed in neutrinos, the power of the SME as a complete effective field theory would permit the testing of such an effect in other processes involving the same coefficients for Lorentz violation. The charged pion decay rate would have been a useful candidate [135].

6. Conclusions

In this work, we have briefly reviewed the history of Lorentz symmetry and its role in modern physics. Some pedagogical examples have been provided to illustrate the meaning of symmetry in physics as well as how a violation of spacetime symmetry would look, and the connection between Lorentz symmetry and CPT symmetry has been presented. The topic of Lorentz violation has been studied extensively through theoretical, phenomenological, experimental, and observational work in the context of the effective field-theory framework of the SME. Following discussion of the motivation for such a framework, we consider the construction of the SME, its popular limits, and theoretical investigations of Lorentz violation. The final section of the paper considers experimental and observational work. Discussion of how tests are performed, the philosophy of broad search, the scope of current limits and open coefficient space, as well as some special aspects of the neutrino sector are considered.

Though much work has been done, a large space of opportunities remains for additional investigation. The types of work that remain can be divided into three basic classes: investigation at the level of the underlying theory, model building and other investigation at the effective field-theory level, and phenomenological, experimental, and observational work performing tests within the SME framework. Work at the underlying-theory level involves considering ways in which Lorentz violation might arise in various candidates for the underlying theory, such as the examples considered in section 2.3, as well as perhaps considering how Lorentz violation at that level might be connected with low-energy physics. At the level of effective field theory, work remains in building additional example models, such as the bumblebee models, that generate the coefficients for Lorentz violation in the SME as a result of spontaneous symmetry breaking. Additional theoretical properties of Lorentz violation in the SME, such as those highlighted in section 4.3, also remain to be explored. Finally, as noted in section 5.4, large sections of coefficient space remain unexplored experimentally. In

some cases, additional phenomenology remains to identify relevant methods of search for such coefficients in various systems. In a number of cases, proposals exist that await additional experimental and observational work.

The development of the SME has provided a systematic approach to exploring Lorentz and CPT violation in nature, and the large amount of work that has been done places the notion of relativity on an ever-stronger foundation. The astoundingly sensitive explorations of Lorentz symmetry that have been done and the additional tests that are possible offer the opportunity to probe Planck-scale physics with existing technology. Though no compelling evidence for Lorentz violation has been found, the large segments of unexplored coefficient space and the remaining theoretical questions suggest that investigations of Lorentz symmetry will continue to play a key role in the ongoing search for Planck-scale physics.

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